:uwait University	Math 102	Date: August 3, 1996
)ept of Math & Comp Sci	Final Examination	Duration: 2 hours

ıswer all of the following questions. Calculators, Pagers and mobile telephones JOT allowed.

(a) Let $f(x) = \dfrac{e^x}{1+e^{2x}}$, $x \geq 0$. Show that f^{-1} exists and state its domain and range and compute $f^{-1}(x)$. (4 points)

(b) Express $\tan(\frac{1}{2}\sin^{-1} x)$, $-1 \leq x \leq 1$ as an algebraic expression in x. (4 points)

Evaluate the following integrals (5 points each)

(a) $\displaystyle\int \sec^{-1} x \, dx.$

(b) $\displaystyle\int \frac{x^5}{\sqrt{1+x^2}} dx$

(c) $\displaystyle\int \frac{3x^3+5x^2+4x+2}{x^2(x^2+2x+2)} dx.$

(d) $\displaystyle\int \frac{1}{1-\sqrt{1-x^2}} \, dx.$

(a) Find $\displaystyle\lim_{x \to 1^+} (\ln x)^{x-1}$, if it exists. (4 points)

(b) Determine whether the integral $\displaystyle\int_1^{\infty} \frac{1}{x^2-2x+5} dx$ is convergent or divergent, and if convergent find its value. (4 points)

(a) Change the polar equation $r = \dfrac{3}{2+\cos\theta}$ to rectangular coordinates and sketch the graph of the resulting equation. (4 points)

(b) Find the area inside <u>both</u> of the polar curves $r = \sin\theta$ and $r = 1-\sin\theta$. (4 points)

(a) Do the lines

$$\begin{aligned} l_1 &: \quad x = 1+t, \quad y = -1-t, \quad z = -4+2t \\ l_2 &: \quad x = 1-u, \quad y = 1+3u, \quad z = 2u \end{aligned}$$

intersect? If so find the point of intersection. (3 points)

(b) Find the equation of the plane determined by the points $P(1,-1,2)$, $Q(0,3,-1)$ and $R(3,-4,1)$. (3 points)

Hossam Ghanem حسام غانم